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Range/Payload Trade-Offs for Ballistic and Cruise Missiles

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DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

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ABSTRACT

Australia is a member of the Missile Technology Control Regime (MTCR), a group of western countries that manufacture and export missile technology. The MTCR was established in 1987, by the seven major western suppliers of missile technology, to control the availability of rocket technology to customer countries. The MTCR sets Guidelines to control the export of missile technology which could be used in delivery systems for weapons of mass destruction. The Guidelines presently provide for a strong presumption of denial for export of ballistic or cruise missiles capable of delivering a 500 kg payload to a range of 300 km and for a less rigorous restriction of 0 kg/300 km systems. There has been much discussion of methods for extending the scope of these criteria to allow for range/payload trade-offs which may be used to adapt a missile, satisfying existing criteria, so that it violates their intention. The method should not, however, be such as to include missiles which cannot provide the necessary performance. This short note outlines work done in Guided Weapons Division of DSTO to support Australian submissions on this topic. Single stage and multi-stage ballistic missiles and cruise missiles are discussed.

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CONTENTS

FIGURES

TABLES

Range/Payload Trade-offs for Ballistic and Cruise Missiles

Executive Summary

The Missile Technology Control Regime (MTCR) was originally established in 1987 by seven western countries with significant manufacturing and technological capabilities in missile technology. The aim of the MTCR is to limit capabilities of less technologically advanced countries in using weapons of mass destruction (WMD) by restricting the availability of missiles and missile technology which might be used to deliver such weapons. The original intention was aimed mainly at systems with nuclear warheads, but more recently the scope has been extended to include both chemical and biological systems.

The original consortium sought to enhance the effectiveness of the controls by expanding the membership and a number of other suppliers were invited to join, including Australia. As a result Australia joined the MTCR in August 1990

The MTCR controls missile technology by setting guidelines to cover aspects of missile technology which could contribute to systems for delivering WMD. The member countries agree to control the export of technology covered by the guidelines. The guidelines initially provided for a strong presumption of denial of export licensing for ballistic and cruise missile systems capable of delivering at least a 500 kg payload to a range of at least 300 km. Subsystems and technology which could be used in such systems was also described and controlled although dual use items could be exported if there was reasonable evidence that the end use was not weapon oriented. Control was also exercised less stringently on systems with a 0 kg/300 km capability.

In a continuing effort to improve its structure and functions, the MTCR members undertook a detailed review of the guidelines covering controlled items. A major aspect of this, where Australia made significant contributions, was concerned with the question of range/payload trade-offs. The MTCR wished to allow for systems with a range payload capability which in their standard form did not violate the guidelines, but which might be simply modified to do so. For example, a single stage missile capable of delivering 200 kg to 600 km did not violate the guidelines in their original form but the payload could be simply increased to 500 kg and the missile might still achieve a range of 300 km. As part of an Australian proposal to develop simple methods for assessing such cases, methods were developed to deal with such situations for a range of ballistic and cruise missiles.

The methods and results of an investigation into simple methods of analysing range/payload trade-offs for ballistic and cruise missiles are described in this report. The results show that for the majority of cases simple methods will show whether range/payload trade-offs leading to violation of MTCR guidelines are possible. For a small percentage of cases, results will be indeterminate. Consideration may have to be given to using more detailed computer based models in these cases or other information which shows that end use is not weapon oriented, since the MTCR seeks not to interfere with legitimate defensive systems or systems intended for scientific investigation.

1. INTRODUCTION

In August 1990 Australia joined the Missile Technology Control Regime (MTCR). The MTCR was established in 1987 by seven major western suppliers of missile technology. The original members have sought to expand participation in order to increase the effectiveness of the controls, and subsequently a number of other countries, including Australia have been invited to join.

The MTCR sets Guidelines to control the export of missile technology which could contribute to weapon delivery systems. The Guidelines initially provided for a strong presumption of denial of export licensing for complete ballistic and cruise missile systems which are capable of delivering at least a 500 kg payload to a range of at least 300 km. Subsystems and technology which can be used in such systems are also controlled, although export licences may be granted if there is reasonable evidence that the end use is **not** weapon oriented. In addition complete systems capable of delivering a 0 kg payload to a range of at least 300 km are also controlled unless there is reasonable evidence that the end use is **not** weapon oriented.

One of the ongoing activities of the MTCR is to review and upgrade the Guidelines. One particular suggestion from Australia was received with some interest by the other members of the MTCR. Australia suggested a modification of the basic criterion which defines missiles controlled by the MTCR. The criteria originally required that the total system should be controlled if it had **both** a minimum range capability of at least 300 km **and** a minimum payload capability of at least 500 kg. However, there are missile systems that fail one of these criteria, but may still be simply modified by alterations to the payload to meet the conditions. For example, a single stage missile of average efficiency which carries a payload of 200 kg to 600 km is easily capable of carrying 500 kg to 300 km, but would not fall within the scope of the criteria. The Guidelines have been changed to allow for the possibility of range/payload trade-offs which may enable systems, which ostensibly satisfy the requirements, to be simply changed into systems which violate the intention of the Guidelines. Methods of assessing the potential for such trade-offs with regard to particular systems are therefore of some interest, and there is discussion from time to time amongst MTCR members with regard to implementing some standard assessment methods. This report deals with some proposed approaches to the problem of assessing the effects on range of changes to payload mass.

Basic theory which can be used to make simple assessments of the effects of range/payload trade-offs for single stage ballistic missiles, multi-stage ballistic missiles and cruise missiles is outlined here. The theory is augmented with some results from a computer based particle trajectory model. The theory and results have been used to provide advice to the Australian delegates to MTCR technical meetings, as an aid in contributing to the development of range/payload trade-off proposals for the MTCR Guidelines.

2. SINGLE STAGE BALLISTIC SYSTEMS

In this chapter, we look at a simple theory involving thrust, range, payload and throw weight for a single stage ballistic missile [1]. A single stage ballistic missile is considered as a missile having only one motor which is used to launch the missile and burns for about 50 s only. During the rest of its flight the missile will be totally unpowered so that its trajectory can be predicted, except for small perturbations, from its velocity and direction at all burnt conditions for the motor. Much of the missile flight will be outside the atmosphere so that simple vacuum trajectory theory can be used to predict the flight. Some aerodynamic means may be used to provide small terminal corrections to improve the terminal accuracy of the missile flight, but in general the missile will be totally uncontrolled once all the fuel is burnt. The missile considered when example values are required for various parameters is a Short Range Ballistic Missile (SRBM) based on the Scud missile, which is a good example of the type of missile and the level of technology at which the MTCR is aimed. The theoretical results are used to develop some simple alternative methods for characterising the range/payload trade-off situation. The results of the simplified theories are compared with the output from a simple computer based model of a single stage ballistic missile. The principal difference between the theoretical solution and the computer based solution is the inclusion of typical aerodynamic drag representations.

In order to simplify the problem sufficiently to obtain a solution without recourse to the computer, two central basic assumptions are made. The advantage of such a solution is that it enables us to isolate the parameters which have a significant effect on the range/payload capability. The two assumptions are:-

- That aerodynamic drag does not significantly affect range predictions. If the motor burn time is sufficiently long that the missile does not achieve too high a velocity until it has cleared the denser part of the atmosphere, aerodynamic drag effects are minimised during the thrust phase. Since the majority of the unpowered part of the trajectory takes place outside the atmosphere the unpowered phase is not significantly affected by drag. Ways of compensating for this assumption are discussed later in the text.

- That launch is near vertical and that the majority of the thrusting phase is close to the vertical. At the end of the thrusting period the elevation for maximum range performance is about 45°, but during most of the thrusting phase of the trajectory the elevation is much higher.

A number of other less important assumptions have been made, which will not materially affect the accuracy of the results. These include such things as ignoring earth curvature, ignoring variation of gravitational force with altitude, and ignoring the effects of the earth's rotation.

Since most of the trajectory is exo-atmospheric for the systems considered here, the range performance can be represented adequately by the in-vacuo result,

$$R = \frac{V_0^{\,2} \sin 2\gamma_0}{g}, \tag{1}$$

where R is range, V_0 is velocity at the commencement of the unpowered stage of the trajectory, that is, at all burnt, γ_0 is the elevation angle at the commencement of unpowered flight, and g is the acceleration due to gravity. Maximum range occurs at $\gamma_0 = 45^o$. Since we are only concerned with range capability, it will be assumed throughout the rest of the derivations that we are dealing with maximum range conditions, and that the launch elevation has been chosen appropriately. Therefore we assume that

$$R = \frac{V_0^2}{g}. \tag{2}$$

The other aspect of the problem is to define the effects of the motor and the thrusting or powered stage of the trajectory. The missile fuel can be characterised by the specific impulse, I_{sp}, which is characteristic of any fuel and oxidant. Information on typical fuel characteristics like the specific impulse and descriptions of other motor characteristics can be found in reference 2. The thrust, T, the mass of fuel, m_f, and the burn time, t_b, can be related, using the specific impulse, by the equation,

$$T = \frac{m_f I_{sp}}{t_b}, \tag{3}$$

We assume that the fuel mass is consumed at a constant rate so that the level of thrust remains constant throughout the powered flight. We also assume that the powered stage starts nearly vertical in direction, and concludes with the elevation of the trajectory near 45^o so that velocity lost due to gravitation during the thrust phase is closely approximated by $0.85gt_b$. Then the velocity at all burnt, when the time, $t = t_b$, is given by

$$V = I_{sp} \ln\left[1 + \frac{m_f}{m_s + m_p}\right] - 0.85gt_b, \tag{4}$$

where m_s is the mass of the structure of the missile, that is, all of the launch mass which is neither fuel nor payload, and m_p is the payload mass. The total $m_s + m_p$ may be characterised as the throw weight, since it is the missile mass which remains after all the fuel is consumed.

Thus equations (1) to (4) define a simple model for a single stage ballistic missile. This model is used in the first sub-section below to develop a methodology for treating range/payload trade-offs. The effects of drag are lumped in with the effects of the structural mass required

to support the payload and maintain the integrity of the missile system. The resulting theory is only approximate, but the overall accuracy of the approach is sufficient for its intended purpose, particularly when drag compensations are used. The great advantage of the approach is its simplicity and its minimal data requirements. In the second subsection, the results from this theory are compared with results generated from a simple, computer based particle trajectory model of a single stage ballistic missile.

2.1 Analytical Approach

The basic assumption behind the approach described below is that minimal information is available on the system being considered, probably only the payload and the maximum range. The analysis in this chapter is first developed on that basis. Further analysis is then proposed for systems where more information is available.

The standard range/payload combination against which trade-offs are to be evaluated is defined as $m_p = \mu$ (500 kg), and $R = \rho$ (300,000 m). Then equations (2) and (4) can be used to find the mass of fuel required to achieve this range /payload combination. The mass of fuel is given by the relation,

$$m_f = (m_s + \mu)\left[\exp\left(\frac{\sqrt{g\rho} + gt_b}{I_{sp}}\right) - 1\right]. \tag{5}$$

This equation then allows us to calculate the mass of fuel required to boost the specified payload, μ, to the required range, ρ. The fuel mass is proportional to the throw weight, $m_s + \mu$, and the proportionality factor is a function of the range and the efficiency of the fuel as characterised by the specific impulse.

Table 1. Nominal Parameter Values

Parameter	Nominal value	Units
g	9.8	$m\,s^{-2}$
I_{sp}	2750	$N\,s\,kg^{-1}$
t_b	50	s
ρ	300	km
μ	500	kg
m_s	600	kg
	1100	kg

All the parameters in equation (5) can be assigned typical values except for m_s, the structural mass of the missile. In fact, as we shall see later, the solutions which are obtained are not particularly sensitive to the values chosen for the other parameters. The only unknown of significance is the structural mass, m_s. The procedure is to

choose a range of values for structural mass, which are typical of single stage ballistic missiles with performance capabilities in the region of interest. Then equations (2), (4), and (5) are used to calculate the effects of range/payload trade-offs. The range of structural mass values varies from as high as 50% of the launch mass for small solid fuel rocket motors of about 250 mm diameter, down to as low as 10% of the launch mass for large rocket motors with diameters greater than 1000 mm. For medium range missiles of the sort discussed here, that is, missiles which are capable of achieving the standard performance of propelling a 500 kg payload to 300 km range, the structural mass of the motor is generally between 30% and 40% depending on the efficiency of the fuel and so on. For consistency with the 500 kg payload, we have chosen to consider the extremes of the structure mass as 600 kg and 1100 kg. These values are quoted in Table 1. The values assigned to the other parameters are also given in Table 1. The resulting range/payload trade-offs which are possible are shown in figure 1.

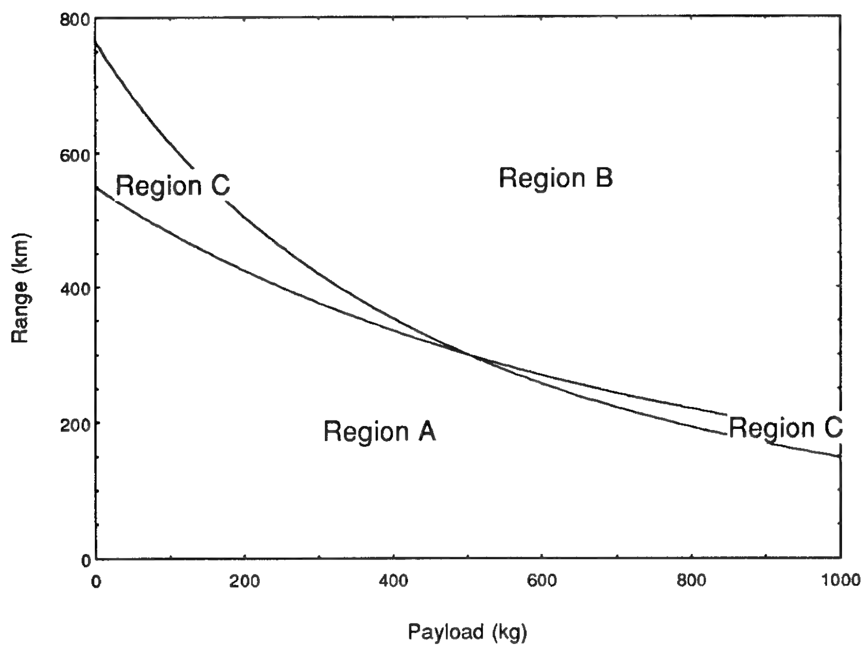

Figure 1. **Range/Payload Trade-off for a Single Stage Ballistic Missile**

By using the above theory and the parameter values quoted in Table 1, range/payload variations can be calculated using the 300 km/500 kg reference point. For a system with a 600 kg structure mass, there is more sensitivity to payload variations, so that the curve extending from nearly 800 km range at 0 kg payload down to 150 km range at 1000 kg payload represents the variation in performance for such a system. The other curve in the figure extending from 550 km down to 200 km range represents a system with a structure mass of 1100 kg, a less efficient system, but a less sensitive one.

The area of uncertainty is the area between the two curves in figure 1, described as region C. Systems which fall in Region B are all excluded because they can always be modified to achieve the proscribed range/payload performance. Systems that fall in Region A on the other hand will not be able to be simply modified to achieve the proscribed performance. Systems falling in Region C, however, are not clearly in one set or the other and may possibly be able to be modified to achieve the proscribed performance. More detailed assessment of such systems would be necessary to determine whether or not range/payload trade-offs could be carried out with such a system. Alternatively, they might be denied on the premise that the possibility of trade-off exists. The region of uncertainty can be altered by alterations to the range of possible values for the structural mass. This problem area will be discussed further subsequently. Other parameters will have a small effect on the size of the area in Region C, but the parameter with the major effect on this is the structural mass component, m_s.

Another factor which affects the area of uncertainty, region C in the figure, is the effect of drag. The effects of aerodynamic drag have been ignored in the treatment used to derive figure 1. However , for the purposes of the simplification of the treatment, the drag can be considered as lumped into the parameter for the structural mass, since the effect on the range/payload trade-offs is very similar. This works well for single stage ballistic missiles. Unfortunately, while it is possible to extend the treatment to multi-stage vehicles, it is relatively complicated. Therefore a slight modification of the above equations has been used to account for the effect of drag more directly. This modification enables the method to be simply extended to cover multi-stage systems as well. The approach is described below.

The approach is based on characterising the effects of drag as an overall energy loss for the trajectory. Equation (2) shows that the range achieved is proportional to the square of the velocity at all burnt, or the kinetic energy, $E_0 = \frac{1}{2} m_1 V_0^2$. The range equation then becomes

$$R = \frac{2E_0}{m_1 g}. \tag{6}$$

If we characterise the effects of drag as an energy loss this translates directly to a proportionate loss of range. A number of sample trajectories were run using the detailed representation of aerodynamic drag in the particle trajectory model outlined in the next subsection. A standard aerodynamic drag curve was estimated for a simple Scud rocket system shape. The drag coefficient as a function of Mach number is shown in figure 2. The percentage energy loss due to aerodynamic drag was calculated by comparing the vacuum trajectory range with the range achieved using the particle trajectory model defined in section 2.2. The comparisons were carried out for the standard 500 kg/300 km payload/range combination over a range of aerodynamic drag values and vehicle structure masses.

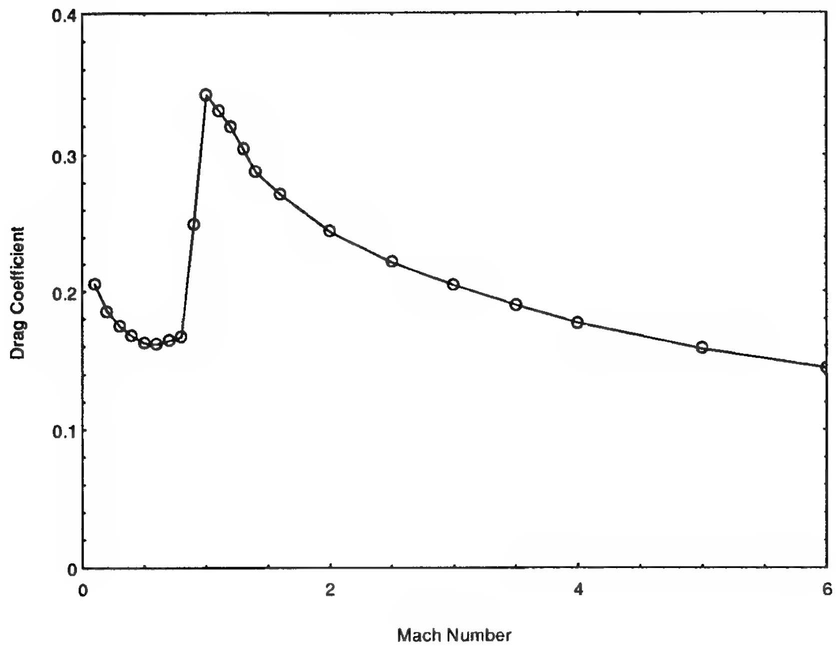

Figure 2. **Drag curve for a single stage SRBM**

The results of the study are shown in figures 3 and 4. Figure 3 shows the drag losses for a system with a structure mass of 600 kg and figure 4 for a system with a structure mass of 1100 kg. The figures show percentage energy lost starting from the vacuum trajectory solution and reducing the range performance to that predicted by the particle trajectory. The three curves in each figure show the results for drag varying from half the estimated drag through the estimated drag to twice that value. The drag losses in each case are approximately constant with regard to variations in payload, but vary considerably with the level of drag and with the structure mass, values from 10% up to 50% are possible. However, for the standard aerodynamic drag values a value of about 25% is realistic.

We now consider the effect of taking drag into consideration when developing the curves shown in figure 1. Figure 5 has been generated using the same data as figure 1 except that an energy loss of 25% due to aerodynamic drag has been allowed for. The changes to the curves defining the separate regions in the figure are generally quite small. The largest changes occur at the extremities of each curve where the ranges vary by about 5% from those in figure 1. The result in each case is to decrease the excursion of the curve from the standard conditions. This means that the inclusion of aerodynamic drag tends to decrease the changes in range which occur due to payload changes from the standard conditions. Thus the size of the region of uncertainty, region C, is reduced slightly.

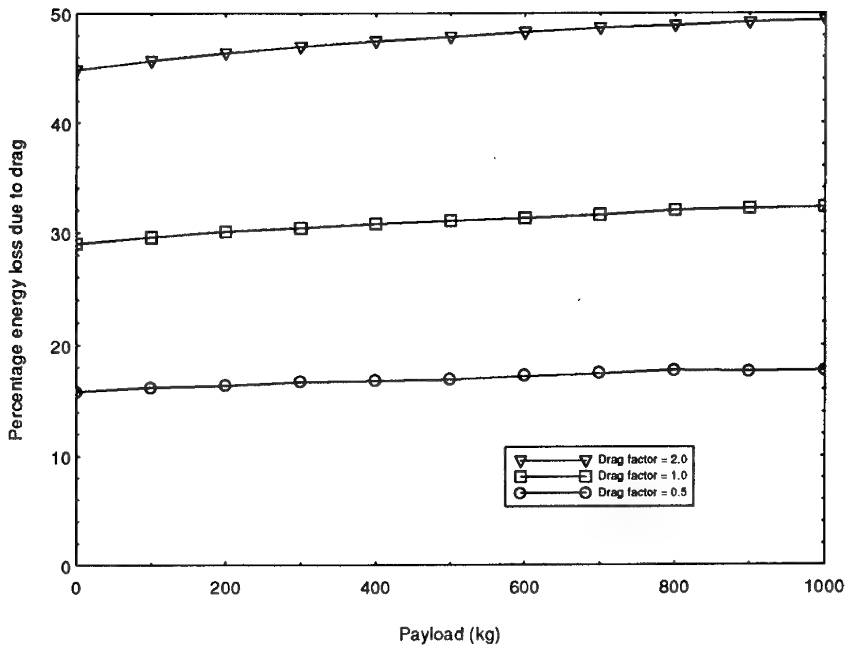

Figure 3. **Energy loss due to aerodynamic drag for a 600 kg structure mass**

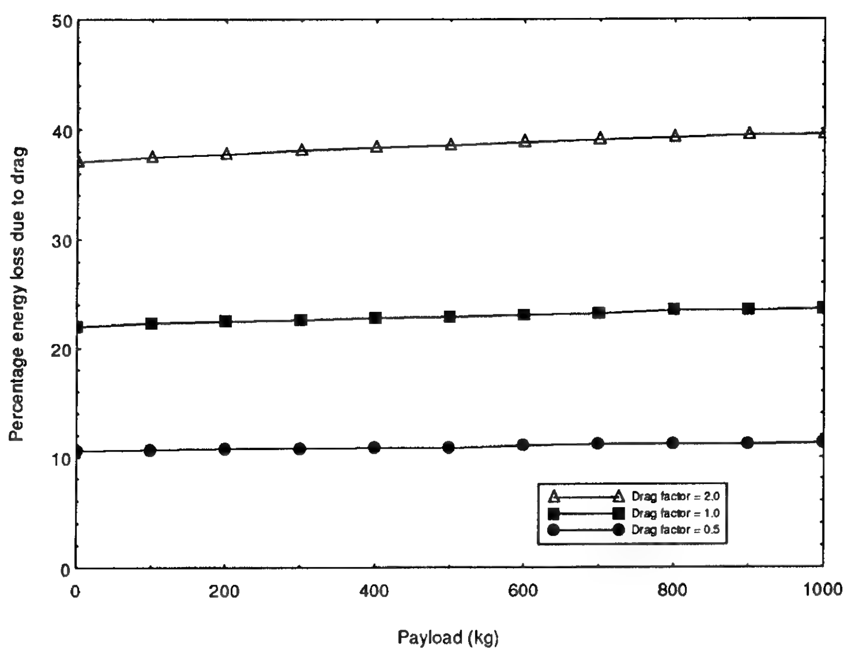

Figure 4. **Energy loss due to aerodynamic drag for a 1100 kg structure mass**

Actual data or conservative estimates can be used for the specific impulse of the fuel and the burn time of the motor, without introducing significant inaccuracies. The safest approach for burn time is to use the burn time which produces optimum range performance, and this can be fairly easily determined for a range of systems using the particle trajectory model outlined below. Specific impulse for the propellant can be approximated by using a conservative value, or if the type of propellant is known by using typical values for that propellant like those shown in Table 2 (ref. 2).

Practical experience with a variety of existing systems indicates that very few systems actually fall in the indeterminate region. In spite of the situation with poor data and inaccuracies, it is generally quite clear whether systems can be simply modified using payload trade-offs to violate the criteria for the MTCR proscribed systems.

A second more precise approach which avoids the regions marked as C in the above figures is to use either a precise knowledge of the structure mass, m_s, if it is available or the total throw weight of the missile, $m_s + m_p$, rather than the payload only, m_p, as the trade-off parameter. The model equations developed above can then be applied directly to the problem to calculate accurately, the range which is consistent with a specific throw weight.

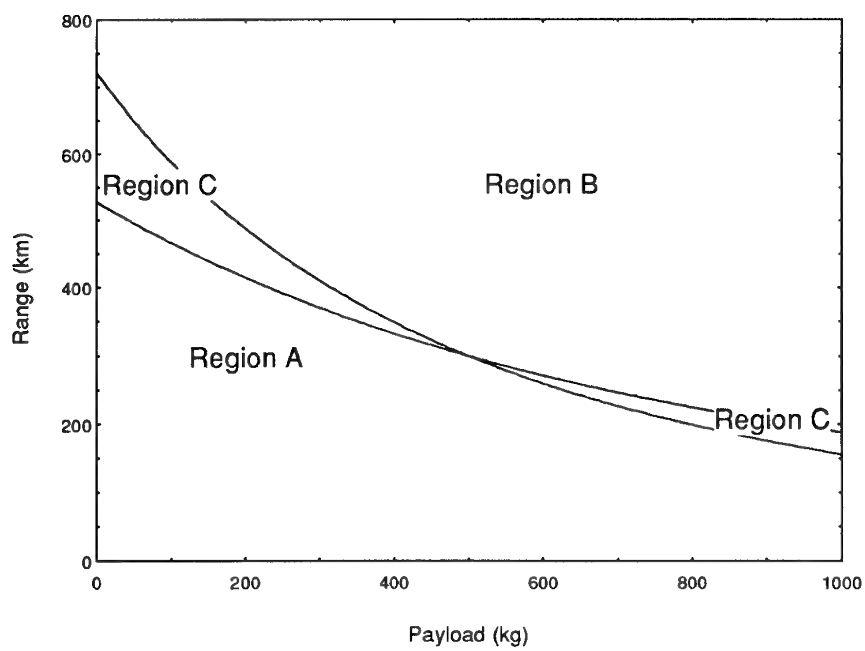

Figure 5. Range/Payload trade-off, including gross drag effects

Table 2. **Specific impulse for classes of propellants**

Propellant Class	Specific Impulse ($N\,s\,kg^{-1}$)
Composite : Ammonium Perchlorate + Binder + Aluminium	2650
Composite : Ammonium Perchlorate + Binder (no Aluminium)	2450
Composite : Ammonium Nitrate + Binder (no Aluminium)	2100
Doublebase : Nitroglycerine/Nitrocellulose with Aluminium	2500
Doublebase : Nitroglycerine/Nitrocellulose (no Aluminium)	2300
Oxides of Nitrogen/Kerosene	2750

2.2 Computer Based Model

The principal differences between the model discussed in this section and that described above lie with the two major assumptions of the previous discussion. It is possible in a simple computer model to make allowance for the aerodynamic drag and also for the change in direction of the thrust vector as the missile turns over on its trajectory during the thrusting stage. The simple computer based model therefore has been used to test out the basic assumptions of the previous discussion.

The equations of motion of the missile are as follows,

$$m\ddot{x} = (T - D)\cos\gamma \,, \tag{7}$$

$$m\ddot{z} = (T - D)\sin\gamma - mg \,, \tag{8}$$

where x and z represent range and altitude coordinates, γ is the elevation of the velocity vector and D is the aerodynamic drag force, defined by the relation,

$$D = \tfrac{1}{2}\rho V^2 S C_D \,, \tag{9}$$

where ρ is the local air density, V is the current velocity of the missile relative to the air, S is the cross-sectional area of the missile, and C_D is the aerodynamic drag coefficient which is a function of the local Mach number.

As we have already discussed figure 2 shows a typical curve for an SRBM, defining the variation of drag coefficient with Mach number for a typical Scud shaped missile. The missile used to derive this drag curve had a body diameter of 0.8 m for a cylindrical

body 8.65 m long. The nose was 2.14 m long, composed of a slightly blunted cone with a semi-angle of 10.5°. Stabilising fins with a 60° sweep, mean chord of 1.6 m and semi-span of 0.75 m with a wedge shaped leading edge. This drag curve was used to investigate the effects of drag on the predictions. The curve shown was used as a representative value and twice this value was used to represent a high drag version, while a 50% reduction was used to represent a low drag version. The effect of drag is not significant in the context of range/payload trade-offs from a standard range/payload condition provided that the motor burn time is chosen for near optimal performance. However, absolute effects can be quite large. As figure 3 shows for high drag systems the energy or range loss due to aerodynamic drag effects can be as high as 50% for a single stage missile, although it is much more likely to be about 25%. For multi-stage missiles the drag losses can be as low as 5%, since in general only the first stage is significantly affected by aerodynamic drag.

Table 3. Comparison of Range Estimates

Drag factor	m_s *(kg)*	m_p *(kg)*	R_c *(km)*	R_a *(km)*
0.5	600	0	735	765
0.5	600	500	300	300
0.5	600	1000	153	148
0.5	1100	0	537	549
0.5	1100	500	300	300
0.5	1100	1000	185	182
1.0	600	0	720	765
1.0	600	500	300	300
1.0	600	1000	157	148
1.0	1100	0	531	549
1.0	1100	500	300	300
1.0	1100	1000	188	182
2.0	600	0	700	765
2.0	600	500	300	300
2.0	600	1000	162	148
2.0	1100	0	522	549
2.0	1100	500	300	300
2.0	1100	1000	191	182

A number of cases were run with the computer model and the results compared with the output from equations (2) to (5). The results were presented in graphical form in figures 3 and 4 and discussed in some detail in the previous section. The results are presented in another form in Table 3.

The table shows both results for range performance from the computer runs, R_c, and comparable results from the analytic expressions of the previous section, R_a. The results show the effect of variations in drag, in structural mass and payload mass. The same procedure was used with the computer model as with the equations. The model was used to calculate the mass of fuel required to achieve the standard conditions of 500 kg payload and 300 km range. Using the mass of fuel so calculated, the model was then used to calculate the range for different payload masses. The procedure was then repeated for different values of the other main parameters. In all cases the burn time, t_b, and the launch elevation, γ_0, was adjusted to achieve maximum range. The process was repeated using the analytic formulation, with a standard burn time of 50 s and launch elevation of 45^0. The results were normalised at the 500 kg/300 km condition and then variations calculated for different payloads.

As we discussed in subsection 2.1, the range predictions using the simple theory of that subsection overestimate range performance by between 10% and 50%. However, when both are normalised at the standard conditions as was done for the Table 3, the results are very similar. The simple theory overestimates the range at longer ranges and underestimates at shorter ranges and therefore would tend to exclude slightly more missiles than the computer model. However, as we stated in comparing figures 1 and 5, the major uncertainty in the overall system is introduced by variations in the structural mass of the missile and other variations and inaccuracies are not very significant. In fact comparing the numbers in Table 3, for standard aerodynamic drag coefficients, variations at the extreme values of payload, 0 kg and 1000 kg, are about 6% different for the lighter structure mass and about 3% different for the heavier structure.

The structure mass as a percentage of the total mass of the rocket motor is determined principally by the range required of the missile system. Realistic assessment of this parameter value is the major determinant of the values for the curves in figures 1 and 5. The values of 600 kg and 1100 kg which have been used in the examples discussed here correspond to 27.5% and 34% structure mass as a percentage of total motor mass respectively. This compares with the estimated 32% for a Scud B which has a similar range but larger payload. Since Scud B is representative of low technology systems this means that the range of values covered by the structure mass values, represents low technology systems at the 1100 kg end up to improved systems at the 600 kg end.

3. MULTI-STAGE BALLISTIC SYSTEMS

The trade-off problem for multi-stage vehicles is complex and will require either detailed information on the mass of the vehicle at light up and all burnt for each stage or a very coarse treatment which is based on the fact that multi-stage vehicles tend to perform like very efficiently constructed single stage vehicles. The systems which we wish to consider in the category of multi-stage ballistic systems are the same as for single stage systems, except that

several motors are connected in series and used consecutively to accelerate the payload to a high initial velocity. Hence two methods can be suggested.

The first method is simply to use the same type of approach as for the single stage missile and to generate a figure similar to figures 1 and 5 but with smaller structural mass components, corresponding to more efficient structures and to more efficient aerodynamic drag management. As the developments in the previous section show the effects of drag are quite small, given that we are concerned with normalising at the standard conditions, 300 km range and 500 kg payload. We might use, to generate the bounding curves, values of m_s = 400 kg and m_s = 800 kg. The corresponding figure would appear in the form given in figure 6. For payloads greater than 500 kg, the area of uncertainty, Region C is quite small, but for smaller payloads the area is larger than for single stage systems. These systems are more sensitive to changes in payload for light payloads because of the small structure mass. In particular the structure mass of the final stage can be quite small, so that changes in payload at the lower end can result in very large relative changes to the throw weight which comprises only the structure mass of the final stage and the payload.

A more accurate estimate of the range/payload trade-off possibilities is of course possible if the mass component make-up of each stage of the multi-stage system is available. The velocity increments achieved with each stage can be calculated using an equation of the form of equation (4) where m_f is the fuel mass for the stage, m_s is the structure mass for the motor for that stage and m_p is the total mass of all the other stages, that is all subsequent stages are regarded as payload. If a range/payload capability is available for the system then this can be reconciled with the estimated range according to the theory of section 2 and then the trade-off situation examined to determine whether the MTCR guidelines are breached. A simple percentage energy loss due to drag can be used in the same way as presented for the single stage systems. Alternatively, all the indeterminate effects such as aerodynamic drag can be lumped into a single energy loss parameter which can be determined by comparing the potential range in a vacuum as calculated using equations (2) and (4), with the actual range performance for the stated payload. The calculation can then be repeated for a 500 kg payload using the same loss factor to determine the trade-off situation.

Since the multi-stage systems are undeniably more complex and require closer examination than the single stage systems, the first method outlined above is likely to be only indicative. The likely structure component masses used to generate figure 6 should therefore be such as to generate a relatively large area for region C, the region of uncertainty. The second method should then be used to determine more accurately whether the guidelines are likely to be infringed.

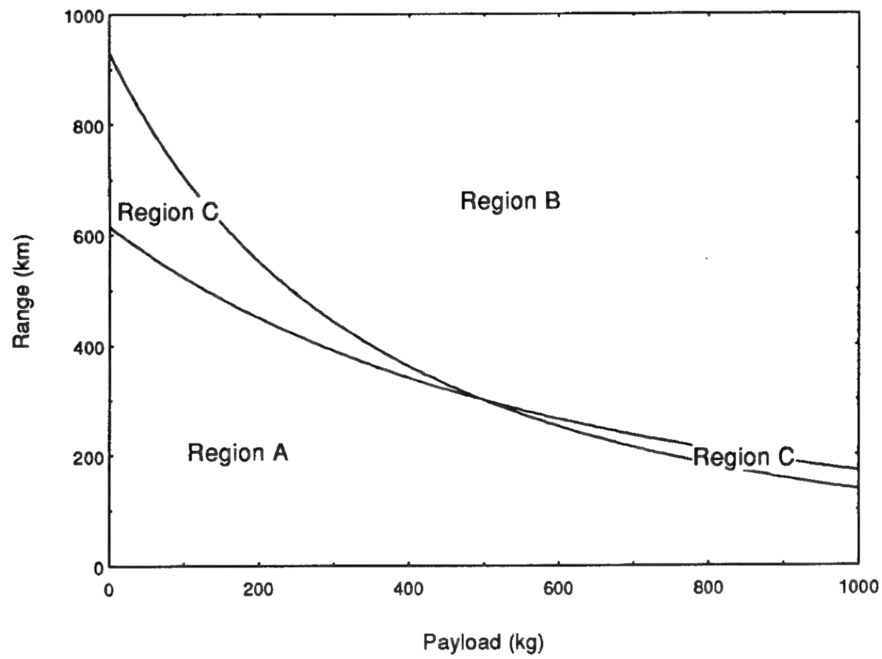

Figure 6. **Range/Payload trade-off for multi-stage systems**

4. CRUISE MISSILES

The cruise missiles which we consider in this section can be characterised essentially by the fact that they fly for the whole of their trajectory within the atmosphere, and have wings which develop lift, to balance the weight of the vehicle. A motor which burns throughout the whole of the flight is used to balance the decelerating effects of aerodynamic drag. The trajectory is essentially horizontal and the velocity is nearly constant throughout most of the flight. Since the flight is within the atmosphere, cruise missile systems often use air breathing propulsion systems such as turbojets or ramjets. The analysis of trade-offs for these missiles is radically different than for ballistic systems. A useful basic text describing atmospheric flight is reference 3. The basic theory developed here to analyse the range/payload trade-offs for cruise missiles is adapted from the treatment in that text.

Solid rocket motors are generally unsuitable for cruise missiles with range capability approaching 300 km range. Current solid rocket motor technology has an upper limit of some 60 to 100 s duration, which limits the achievable ranges to less than 100 km regardless of payload. Examples are Exocet, Penguin and Ikara. Consequently all long range cruise missiles require a more efficient form of propulsion. Turbojets and ramjets are preferred, but liquid propellant rockets (for high speed and limited range) and propellers (for low speed and long range) are feasible. In the following discussion we are therefore not concerned with solid propellant systems.

The theory for a cruise missile is based on the assumption that it flies at constant velocity with lift balancing weight and thrust balancing drag. The thrust is given by

$$T = I_{sp}\dot{m}, \tag{10}$$

and the thrust balances the drag to maintain constant velocity, so that

$$T = \tfrac{1}{2}\rho V^2 S C_D. \tag{11}$$

The missile flies also at near constant altitude. It may fly at different altitudes over the whole trajectory but during most phases the altitude will be constant. Hence the lift must balance the weight of the missile, so that

$$mg = \frac{1}{2}\rho V^2 S C_L, \tag{12}$$

and the air density, ρ, is constant.

We can obtain a relation between thrust and mass of the missile by dividing equation (11) by equation (12) and manipulating the result, so that we find,

$$T = mg\left(C_D/C_L\right). \tag{13}$$

The treatment in reference 3 is based on using a parabolic drag polar, which simply means that the drag varies with the square of the lift in the form,

$$C_D = C_{D_0} + KC_L^2. \tag{14}$$

The equations (10) to (14) can be developed in non-dimensional form to provide differential equations of motion for range and time of flight, using vehicle mass as the independent variable. Reference 3 then develops particular solutions for a variety of cases :-

- constant angle of attack, or constant lift coefficient,
- constant velocity, and
- constant thrust.

The maximum range attained for each condition is very similar. The differences are less than 1% for fuel mass to total launch mass ratios of less than 0.25, and the constant angle of attack solution is the largest. The constant angle of attack solution is also the most analytically tractable, and so we will address this in the following discussion.

The following assumptions are used as the basis for developing range/payload trade-off treatment for cruise missiles:-

- constant altitude flight;
- parabolic drag polar, which is an accurate representation of the relation between lift and drag for typical cruise missile aerodynamic designs;
- constant angle of attack flight, which provides little restriction on the optimisation of the trajectory for maximum range performance.

Under these assumptions, reference 3 shows that the maximum range is given by

$$R_{\max} = \frac{1}{2}\left(\frac{2g}{\rho S}\right)^{1/2}\left(\frac{K}{C_{D_0}}\right)^{1/4}\left(KC_{D_0}\right)^{1/2}\left(\frac{I_{SP}}{g}\right)\ \left[m_0^{1/2} - m_1^{1/2}\right], \tag{15}$$

and this maximum occurs at an angle of attack such that the lift coefficient is given by

$$C_L = \sqrt{\frac{C_{D_0}}{3K}}, \tag{16}$$

which corresponds to a lift-to-drag ratio,

$$\frac{C_L}{C_D} = \frac{\sqrt{3}}{4}\sqrt{\frac{1}{KC_{D_0}}}\ . \tag{17}$$

This lift-to-drag ratio is 86.6% of the maximum value. The maximum value of the ratio occurs at $C_{L_{opt}} = \sqrt{C_{D_0}/K}$, almost twice the value in equation (16).

In order to examine the range/payload trade-off situation, equation (15) is simplified to the following form, by aggregating all the leading constants,

$$R_{\max} = \text{Constant X}\left[m_0^{1/2} - m_1^{1/2}\right], \tag{18}$$

where m_0 is the launch mass composed of structure mass, m_S, payload mass, m_P, and fuel mass, m_F, and m_1 is the final mass when all the fuel is consumed, composed of structure mass and payload mass only. If we know the structure and fuel mass components for any missile the value of the constant can be calculated for the 300 km/500 kg range/payload condition, and equation (18) used to develop similar range/payload curves for cruise missiles to those already developed for ballistic missiles.

As an example of the results which might be expected, figure 7 has been calculated based on a system with 200 kg of fuel and alternative structure masses of 500 kg and 1000 kg respectively. As might be expected, figure 7 shows that there is a lot less variation in range capability with variations in payload for cruise missiles.

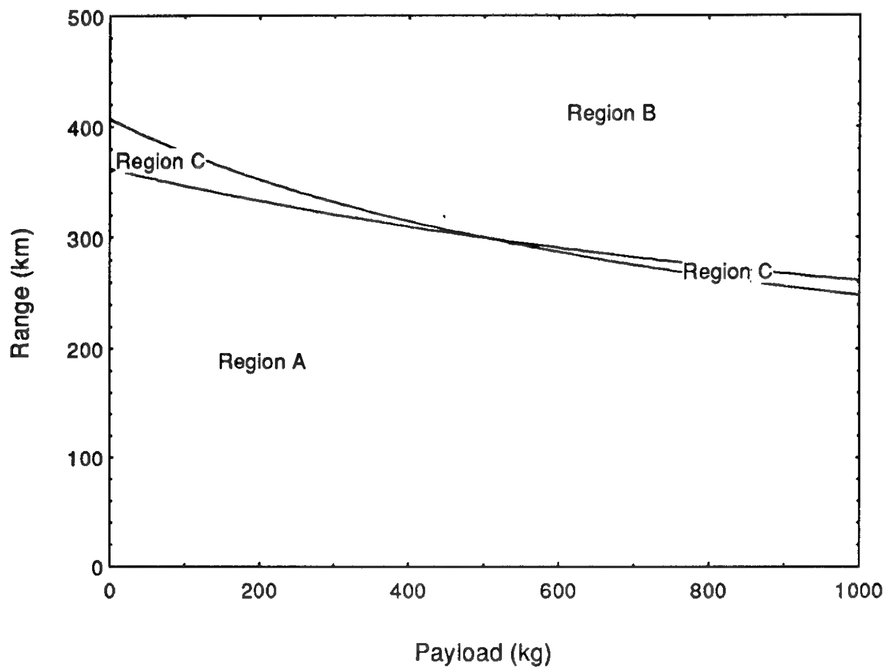

Figure 7. Range/Payload trade-off for a cruise missile

More work needs to be done to examine the trade-off options for cruise missiles. A survey of cruise missile systems is needed to determine realistic values for the different mass components (fuel, structure and payload), for systems whose performance characteristics are in the class where range/payload trade-off considerations are relevant. However, the above discussion shows that there is a firm basis for developing a similar approach to that described for ballistic missile systems.

5. SUMMARY AND CONCLUSIONS

Theoretical treatments for ballistic missiles, both single stage and multi-stage, which enable application of range/payload trade-off criteria have been presented. Some treatment of cruise missiles is also included. The report provides general graphical data which enables first approximation assessments of range/payload capabilities to be carried out with minimal

information. Approaches for utilising small amounts of additional information to make more accurate assessments of range/payload trade-offs are also provided.

For ballistic missiles, both single stage and multi-stage, a breakdown of mass components into fuel, structure and payload for each stage is sufficient to enable a relatively accurate assessment of the trade-off possibilities to be carried out. The effect of aerodynamic drag, whilst it may affect the absolute range capability of a missile by as much as 50%, introduces only small errors into the trade-off calculations as we can see by examination of Table 3. Increased accuracy in the assessment of drag effects can be achieved, if required, by broad introduction of drag correction factors as an overall percentage of energy or range loss due to drag.

The mechanisms which produce the effects of trade-offs on cruise missile performance are rather different in kind from those for ballistic missiles. An example of how the trade-off question can be simply managed for cruise missiles has been presented. It shows that range performance of cruise missiles is much less sensitive to payload variations than for ballistic missiles.

It is important to realise that the approach to examining range/payload trade-offs can be adapted according to the availability of data on the system under consideration. If absolutely minimal data is available, then only the simplest judgements can be made based on the trade-off figures presented in the text. If detailed mass components are known then the theory presented in the text can be used to derive a more accurate answer. If detailed aerodynamic data is available together with some physical data on the mass and geometry of the missile, then computer models can be used to calculate trajectories under a variety of conditions to enable even more accurate assessment of trade-off effects. Therefore a layered approach is suggested.

NOTATION

C_D	Aerodynamic drag coefficient of missile
C_L	Aerodynamic lift coefficient of cruise missile
D	Force due to aerodynamic drag (N)
g	Acceleration due to gravity ($m\ s^{-2}$)
I_{sp}	Specific impulse of missile fuel ($N\ s\ kg^{-1}$)
m	Instantaneous mass of missile (kg)
$\dot{m}$	Rate of fuel consumption (kg/s)
m_f	Mass of fuel at launch (kg)
m_p	Mass of missile payload (kg)
m_s	Mass of missile structure, other than fuel and payload (kg)
R	Range of missile (m)
S	Cross-sectional or reference area of missile (m^2)
T	Thrust level of missile motor (N)
t_b	Total burn time of missile motor (s)
V_0	Velocity of missile at all burnt ($m\ s^{-1}$)
x, z	Range and altitude coordinates (m)

γ_0 Elevation of missile trajectory at all burnt (*rad*)

μ Standard payload at reference range/payload condition (*kg*)

ρ Local air density (*kg* m^{-3})

ρ Standard range at reference range/payload condition (*m*)

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Range/Payload Trade-offs for Ballistic and Cruise Missiles

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16. Abstract Australia is a member of the Missile Technology Control Regime (MTCR), a group of western countries that manufacture and export missile technology. The MTCR was established in 1987, by the seven major western suppliers of missile technology, to control the availability of rocket technology to customer countries. The MTCR sets Guidelines to control the export of missile technology which could be used in delivery systems for weapons of mass destruction. The Guidelines presently provide for a strong presumption of denial for export of ballistic or cruise missiles capable of delivering a 500 kg payload to a range of 300 km and for a less rigorous restriction of 0 kg/300 km systems. There has been much discussion of methods for extending the scope of these criteria to allow for range/payload trade-offs which may be used to adapt a missile, satisfying existing criteria, so that it violates their intention. The method should not, however, be such as to include missiles which cannot provide the necessary performance. This short note outlines work done in Guided Weapons Division of DSTO to support Australian submissions on this topic. Single stage and multi-stage ballistic missiles and cruise missiles are discussed.